# EE 508
# Lecture 22

Sensitivity Functions

- Comparison of Circuits
- Predistortion and Calibration

## Review Correction from last time

Theorem: If all op amps in a filter are ideal, then $\omega_o$, Q, BW, all band edges, and all poles and zeros are homogeneous of order 0 in the impedances.

Theorem: If all op amps in a filter are ideal and if T(s) is a dimensionless transfer function, T(s), $T(j\omega)$, $|T(j\omega)|$, $\angle T(j\omega)$, are homogeneous of order 0 in the impedances

Review from last time

# Bilinear Property of Electrical Networks

Theorem: Let x be any component or Op Amp time constant ($1^{st}$ order Op Amp model) of any linear active network employing a finite number of amplifiers and lumped passive components. Any transfer function of the network can be expressed in the form

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

where $N_0$, $N_1$, $D_0$, and $D_1$ are polynomials in s that are not dependent upon x

A function that can be expressed as given above is said to be a bilinear function in the variable x and this is termed a bilateral property of electrical networks.

The bilinear relationship is useful for

1. Checking for possible errors in an analysis
2. Pole sensitivity analysis

# Root Sensitivities

Consider expressing T(s) as a bilinear fraction in x

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}=\frac{N(s)}{D(s)}$$

Theorem: If $z_i$ is any simple zero and/or $p_i$ is any simple pole of T(s), then

$$S_x^{z_i}=\left(\frac{x}{z_i}\right)\left(\frac{-N_1(z_i)}{\dfrac{dN(z_i)}{dz_i}}\right) \quad \text{and} \quad S_x^{p_i}=\left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\dfrac{dD(p_i)}{dp_i}}\right)$$

Note: Do not need to find expressions for the poles or the zeros to find the pole and zero sensitivities !

Note: Do need the poles or zeros but they will generally be known by design

Note: Will make minor modifications for extreme values for x (i.e. τ for op amps)

# Root Sensitivities

Theorem: If $p_i$ is any simple pole of T(s), then

$$S_x^{p_i} = \left(\frac{x}{p_i}\right)\left(\frac{-D_1(p_i)}{\frac{dD(p_i)}{dp_i}}\right)$$

**Proof** (similar argument for the zeros)

$$D(s)=D_0(s)+xD_1(s)$$

By definition of a pole,

$$D(p_i)=0$$

$$\therefore \quad D(p_i)=D_0(p_i)+xD_1(p_i)=0$$

# Root Sensitivities

$$\therefore \qquad D(p_i) = D_0(p_i) + xD_1(p_i)$$

Differentiating this expression implicitly WRT x, we obtain

$$\frac{\partial D_0(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + \left[ x\frac{\partial D_1(p_i)}{\partial p_i}\frac{\partial p_i}{\partial x} + D_1(p_i) \right] = 0$$

Re-grouping, obtain

$$\frac{\partial p_i}{\partial x}\left[ \frac{\partial D_0(p_i)}{\partial p_i} + x\frac{\partial D_1(p_i)}{\partial p_i} \right] = -D_1(p_i)$$

But term in brackets is derivative of $D(p_i)$ wrt $p_i$, thus

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left( \dfrac{\partial D(p_i)}{\partial p_i} \right)}$$

# Root Sensitivities

$$\frac{\partial p_i}{\partial x} = -\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Finally, from the definition of sensitivity,

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

# Root Sensitivities

$$S_x^{p_i} = \frac{x}{p_i}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{p_i}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

Observation: Although the sensitivity expression is readily obtainable, direction information about the pole movement is obscured because the derivative is multiplied by the quantity $p_i$ which is often complex. Usually will use either

$$\mathcal{S}_x^{p_i} = \frac{\partial p_i}{\partial x}$$

or

$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x} = -\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

which preserve direction information when working with pole or zero sensitivity analysis.

# Root Sensitivities

Summary: Pole (or zero) locations due to component variations can be approximated with simple analytical calculations without obtaining parametric expressions for the poles (or zeros).

$$p_i \simeq p_i\Big|_{\substack{\text{Ideal}\\ \text{Components}}} + \Delta p_i$$

where

$$\Delta p_i \simeq \Delta x \bullet \mathcal{S}_x^{p_i}$$

$$\mathcal{S}_x^{p_i} = -\frac{D_1(p_i)}{\left.\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)\right|_{p_{iN}}}$$

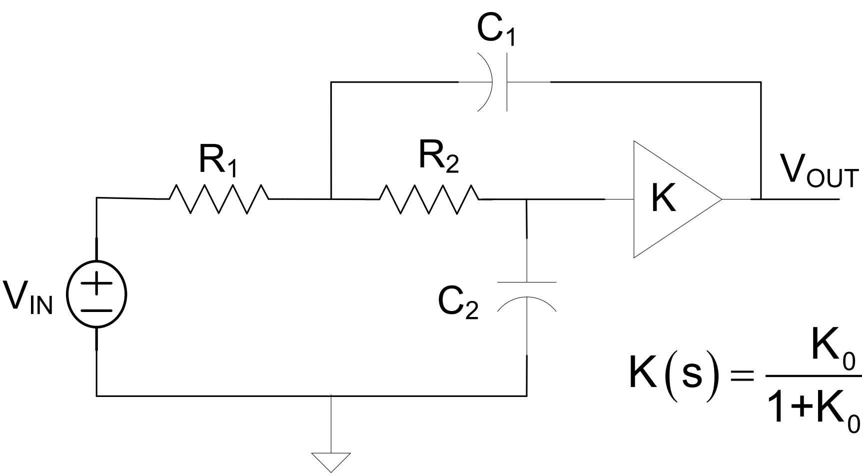
and

$$D(s) = D_0(s) + x \bullet D_1(s)$$

Alternately,

$$\Delta p_i \simeq \left(|p_i| \frac{\Delta x}{x}\right) \tilde{S}_x^{p_i}$$

## Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\dfrac{\partial D(p_i)}{\partial p_i}\right)}$$

$$T(s)=\frac{\dfrac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{(1-K_0)}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_1C_1}+\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]+\dfrac{1}{R_1R_2C_1C_2}\right)}$$

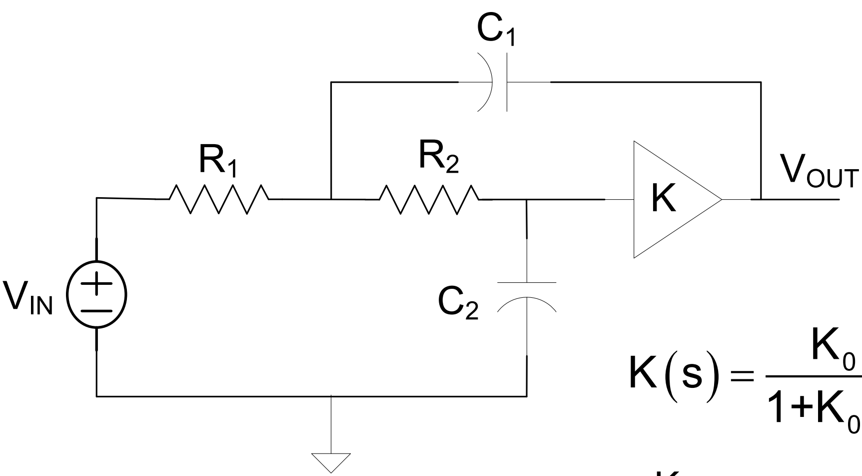
**write in bilinear form**

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+K_0\tau s\left(s\dfrac{1}{C_1}\right)+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1-K_0)}{R_2C_2}+K_0\tau s\left(s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}\right]\right)\right]\right]}$$

**evaluate at τ=0**

$$T(s)=\frac{\dfrac{K_0}{R_2C_1C_2}}{\left(s\dfrac{1}{C_1}+\dfrac{1}{R_2C_1C_2}+\dfrac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\dfrac{1}{R_2C_1}+\dfrac{(1-K_0)}{R_2C_2}\right]\right]}$$

# Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

$$K(s)=\frac{K_0}{1+K_0\tau s}$$

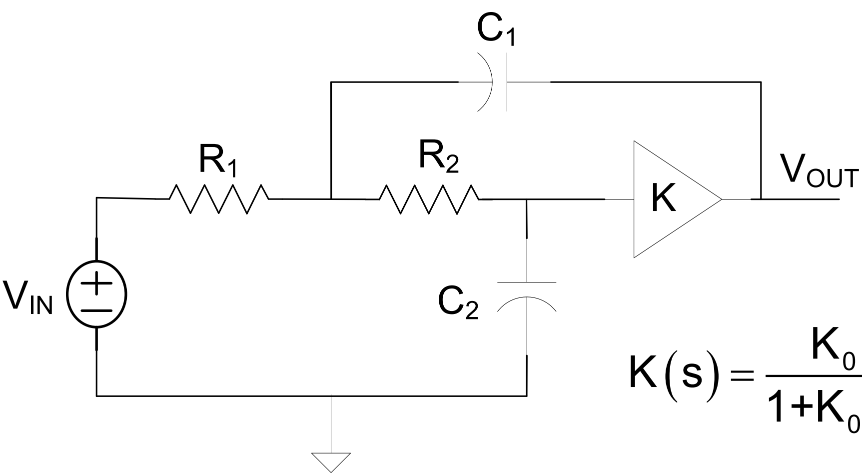
$$T(s)=\frac{\frac{K_0}{R_2C_1C_2}}{\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1\text{-}K_0)}{R_2C_2}\right]\right]}$$

$$D_1(s)=s^2+s\left[\frac{1}{R_2C_1}+\frac{(1\text{-}K_0)}{R_2C_2}\right]$$

$$D(s)=\left(s\frac{1}{C_1}+\frac{1}{R_2C_1C_2}+\frac{1}{R_2C_1C_2}\right)+R_1\left[s^2+s\left[\frac{1}{R_2C_1}+\frac{(1\text{-}K_0)}{R_2C_2}\right]\right]=R_1\left(s^2+s\left[\frac{\omega_0}{Q}\right]+\omega_0^2\right)$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1\text{-}K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

## Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$T(s)=\frac{N_0(s)+xN_1(s)}{D_0(s)+xD_1(s)}$$

T

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{x}{|p_i|}\right)\frac{D_1(p_i)}{\left(\frac{\partial D(p_i)}{\partial p_i}\right)}$$

$$T(s)=\frac{\frac{K_0}{R_1R_2C_1C_2}}{s^2+s\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}}$$

$$p^2+p\left[\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]+\frac{1}{R_1R_2C_1C_2}=0$$

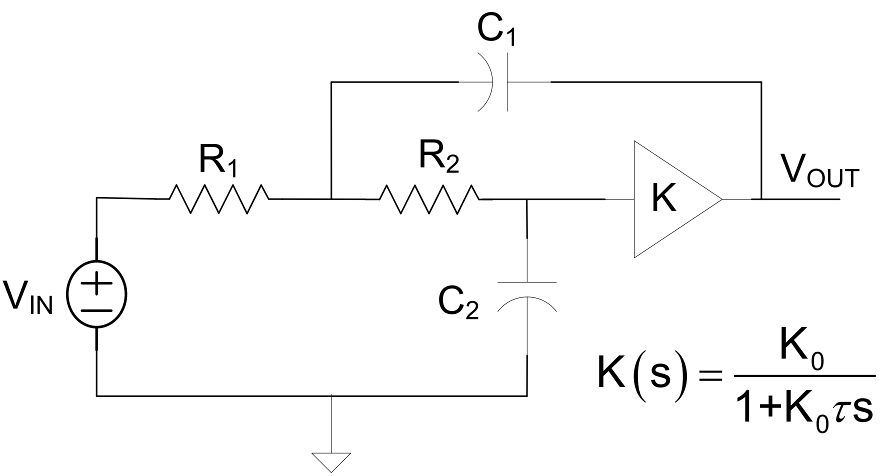
$$p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]=-\frac{1}{R_1R_2C_1C_2}-p\frac{1}{R_1C_1}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=-\left(\frac{1}{|p_i|}\right)\frac{p^2+p\left[\frac{1}{R_2C_1}+\frac{(1-K_0)}{R_2C_2}\right]}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{|p_i|}\right)\frac{\frac{1}{R_1R_2C_1C_2}+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

## Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\frac{1}{R_1C_1}}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

For equal R, equal C $\quad \omega_0 = \frac{1}{RC}$

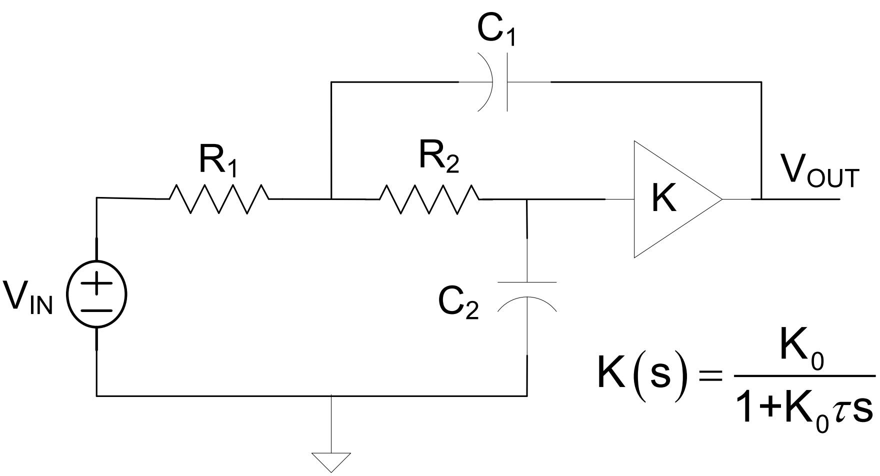
$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\left(\frac{1}{\omega_0}\right)\frac{\omega_0^2+p\omega_0}{\left(2p_i+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{\omega_0 - \frac{\omega_0}{2Q}\pm\frac{\omega_0}{2Q}\sqrt{1-4Q^2}}{\pm\frac{\omega_0}{Q}\sqrt{1-4Q^2}}$$

$$\tilde{S}_{R_1}^{p}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}=\frac{\omega_0+p}{\left(2p+\frac{\omega_0}{Q}\right)}$$

$$\tilde{S}_{R_1}^{p}=\frac{Q-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4Q^2}}{\pm\sqrt{1-4Q^2}}$$

## Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$K(s)=\frac{K_0}{1+K_0\tau s}$$

$$\tilde{S}_x^{p_i}=\frac{x}{|p_i|}\frac{\partial p_i}{\partial x}$$

For equal R, equal C

$$\tilde{S}_{R_1}^{p}=\frac{Q-\frac{1}{2}\pm\frac{1}{2}\sqrt{1-4Q^2}}{\pm\sqrt{1-4Q^2}}$$

Note this contains magnitude and direction information

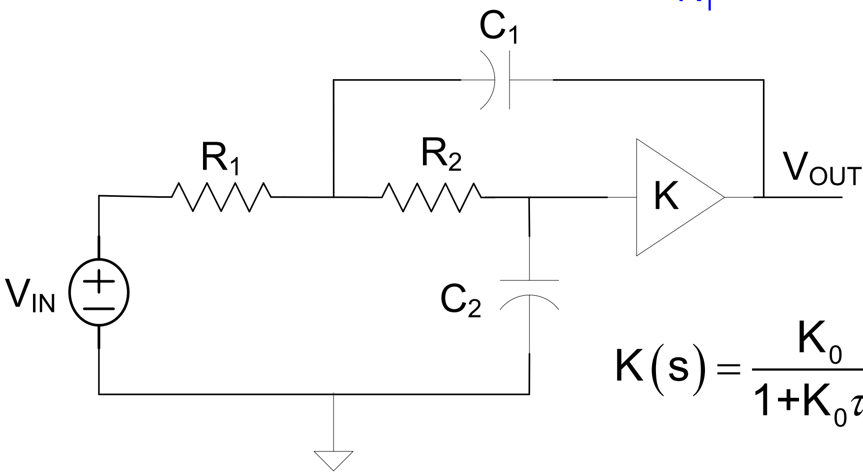
For high Q

$$\tilde{S}_{R_1}^{p}=\frac{Q\pm\frac{1}{2}\sqrt{-4Q^2}}{\pm\sqrt{-4Q^2}}=\frac{Q\pm jQ}{\pm j2Q}=\frac{1\pm j}{\pm j2}=\frac{j\pm 1}{\pm 2}=\frac{1}{2}\pm\frac{1}{2}j$$

$$\Delta p_i \cong |p_i|\tilde{S}_x^{p_i}\frac{\Delta x}{x}$$

$$\boxed{\Delta p_i \cong \omega_0\left(0.5\pm 0.5j\right)\frac{\Delta R_1}{R_1}}$$

## Example: Determine $\tilde{S}_{R_1}^{p_i}$ for the +KRC Lowpass Filter for equal R, equal C




$$\tilde{S}_x^{p_i} = \frac{x}{|p_i|}\frac{\partial p_i}{\partial x}$$

For equal R, equal C

$$K(s) = \frac{K_0}{1+K_0\tau s}$$

For high Q

$$\Delta p_i \cong \omega_0 \left(0.5 \pm 0.5j\right)\frac{\Delta R_1}{R_1}$$

Could we have assumed equal R equal C before calculation?

No ! Analysis would not apply (not bilinear)

Results would obscure effects of variations in individual components

Was this a lot of work for such a simple result?

Yes ! But it is parametric and still only took maybe 20 minutes

But it needs to be done only once for this structure

Can do for each of the elements

What is the value of this result?

Understand how components affect performance of this circuit

Compare performance of different circuits for architecture selection

# Transfer Function Sensitivities

$$S_x^{T(s)}\Big|_{s=j\omega} = S_x^{T(j\omega)}$$

$$S_x^{T(j\omega)} = S_x^{|T(j\omega)|} + j\theta S_x^{\theta}$$ where $$\theta = \angle T(j\omega)$$

$$S_x^{|T(j\omega)|} = \mathrm{Re}\left(S_x^{T(j\omega)}\right)$$

$$S_x^{\theta} = \frac{1}{\theta}\mathrm{Im}\left(S_x^{T(j\omega)}\right)$$

# Transfer Function Sensitivities

If T(s) is expressed as $T(s) = \dfrac{\sum\limits_{i=0}^{m} a_i s^i}{\sum\limits_{i=0}^{n} b_i s^i} = \dfrac{N(s)}{D(s)}$

then

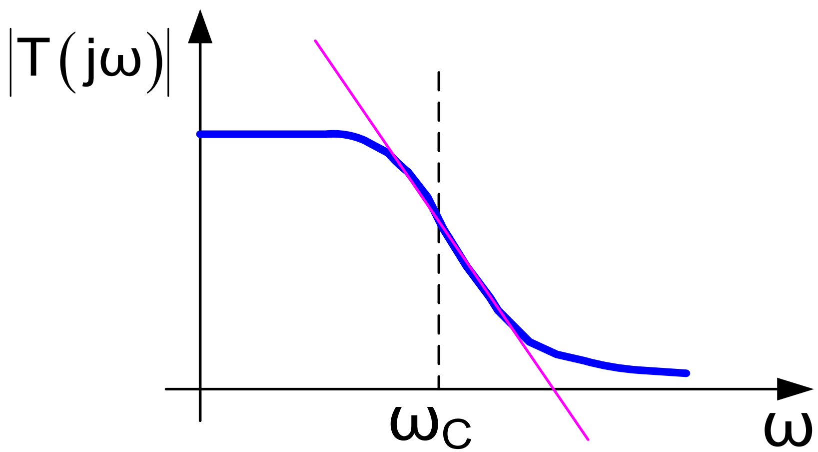
$$S_x^{T(s)} = \frac{\sum\limits_{i=0}^{m} a_i s^i S_x^{a_i}}{N(s)} - \frac{\sum\limits_{i=0}^{n} b_i s^i S_x^{b_i}}{D(s)}$$

If T(s) is expressed as $T(s) = \dfrac{N_0(s) + xN_1(s)}{D_0(s) + xD_1(s)}$

$$S_x^{T(s)} = \frac{x\left[D_0(s)N_1(s) - N_0(s)D_1(s)\right]}{\left(N_0(s) + xN_1(s)\right)\left(D_0(s) + xD_1(s)\right)}$$

# Band-edge Sensitivities

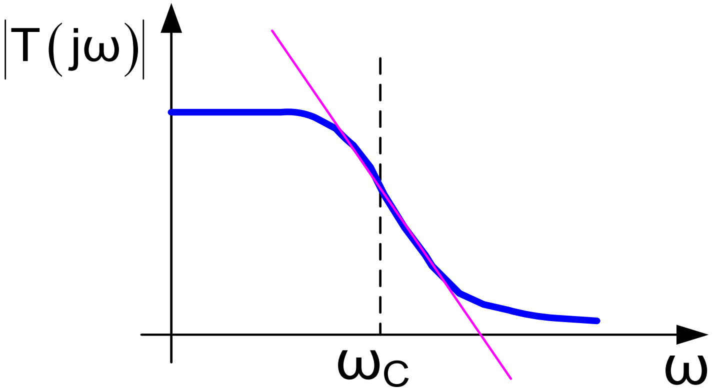
The band edge of a filter is often of interest. A closed-form expression for the band-edge of a filter may not be attainable and often the band-edges are distinct from the $\omega_0$ of the poles. But the sensitivity of the band-edges to a parameter x is often of interest.




Want

$$S_x^{\omega_C} = \frac{\partial \omega_C}{\partial x} \bullet \frac{x}{\omega_C}$$

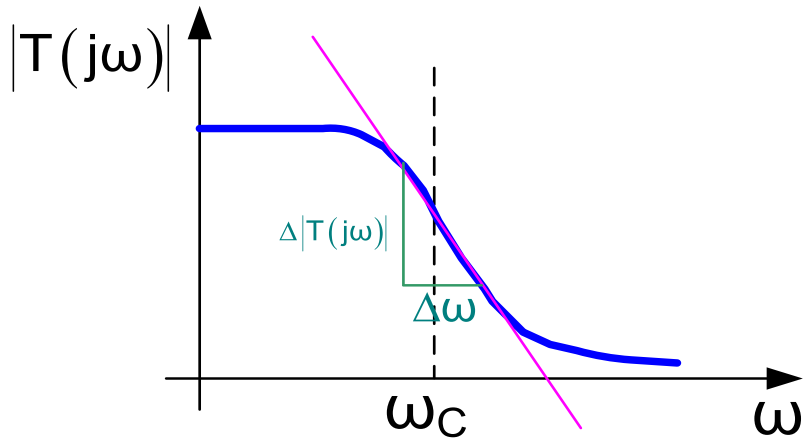
# Band-edge Sensitivities




Theorem: The sensitivity of the band-edge of a filter is given by the expression

$$S_x^{\omega_C} = \frac{\left. S_x^{|T(j\omega)|} \right|_{\omega=\omega_C}}{\left. S_\omega^{|T(j\omega)|} \right|_{\omega=\omega_C}}$$

# Band-edge Sensitivities

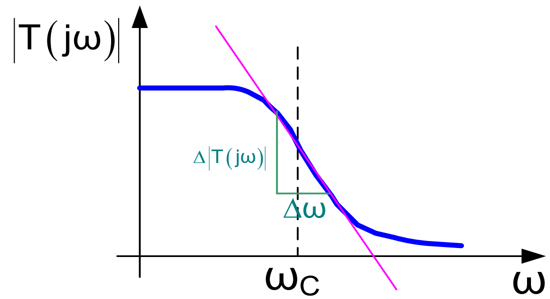



Proof:

Observe
$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \cong \frac{\Delta\left|T(j\omega)\right|}{\Delta\omega}$$

$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \cong \frac{\Delta\left|T(j\omega)\right|}{\Delta x} \bullet \frac{\Delta x}{\Delta\omega} \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x}}{\dfrac{\partial\omega}{\partial x}}$$

# Band-edge Sensitivities




$$\frac{\partial\left|T(j\omega)\right|}{\partial\omega} \cong \frac{\Delta\left|T(j\omega)\right|}{\Delta x} \bullet \frac{\Delta x}{\Delta\omega} \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x}}{\dfrac{\partial\omega}{\partial x}}$$

$$\frac{\partial\omega}{\partial x} \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x}}{\dfrac{\partial\left|T(j\omega)\right|}{\partial\omega}}$$

$$\frac{\partial\omega}{\partial x} \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x} \bullet \dfrac{x}{\left|T(j\omega)\right|}}{\dfrac{\partial\left|T(j\omega)\right|}{\partial\omega} \bullet \dfrac{\omega}{\left|T(j\omega)\right|}} \left(\frac{\omega}{x}\right)$$

$$\frac{\partial\omega}{\partial x} \bullet \left(\frac{x}{\omega}\right) \cong \frac{\dfrac{\partial\left|T(j\omega)\right|}{\partial x} \bullet \dfrac{x}{\left|T(j\omega)\right|}}{\dfrac{\partial\left|T(j\omega)\right|}{\partial\omega} \bullet \dfrac{\omega}{\left|T(j\omega)\right|}}$$

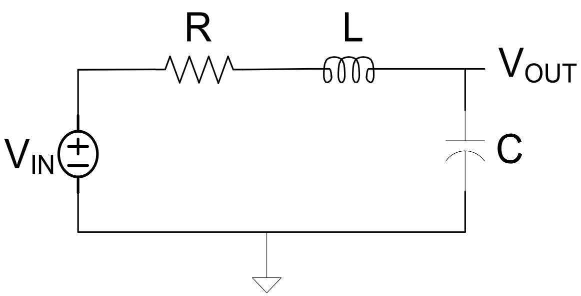
$$S_x^{\omega} = \frac{S_x^{\left|T(j\omega)\right|}}{S_\omega^{\left|T(j\omega)\right|}}$$

$$S_x^{\omega_C} = \frac{\left. S_x^{\left|T(j\omega)\right|} \right|_{\omega=\omega_C}}{\left. S_\omega^{\left|T(j\omega)\right|} \right|_{\omega=\omega_C}}$$

# Sensitivity Comparisons

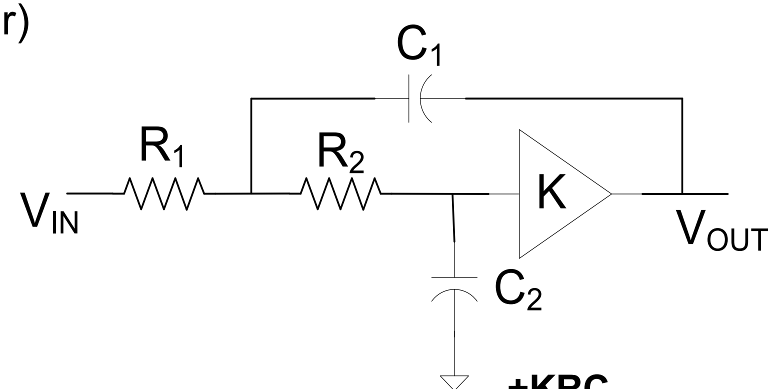
## Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



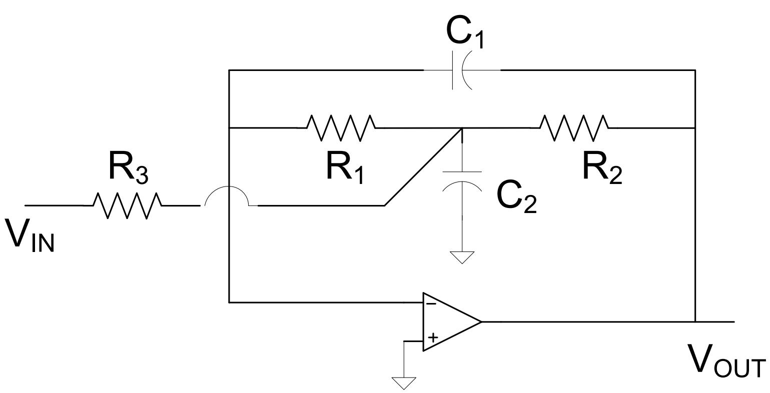

**Passive RLC**

(a)



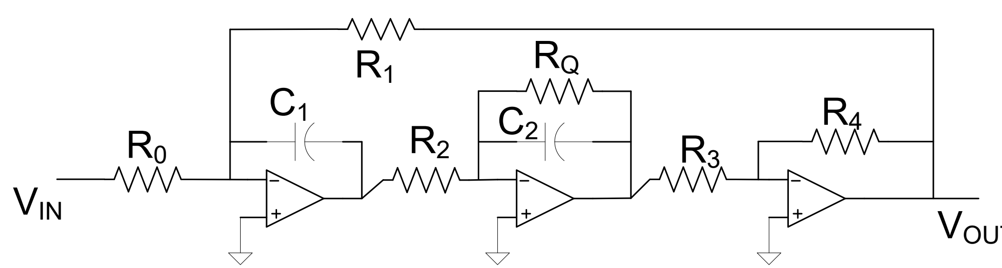

**+KRC**

(b)




**Bridged-T Feedback**

(c)



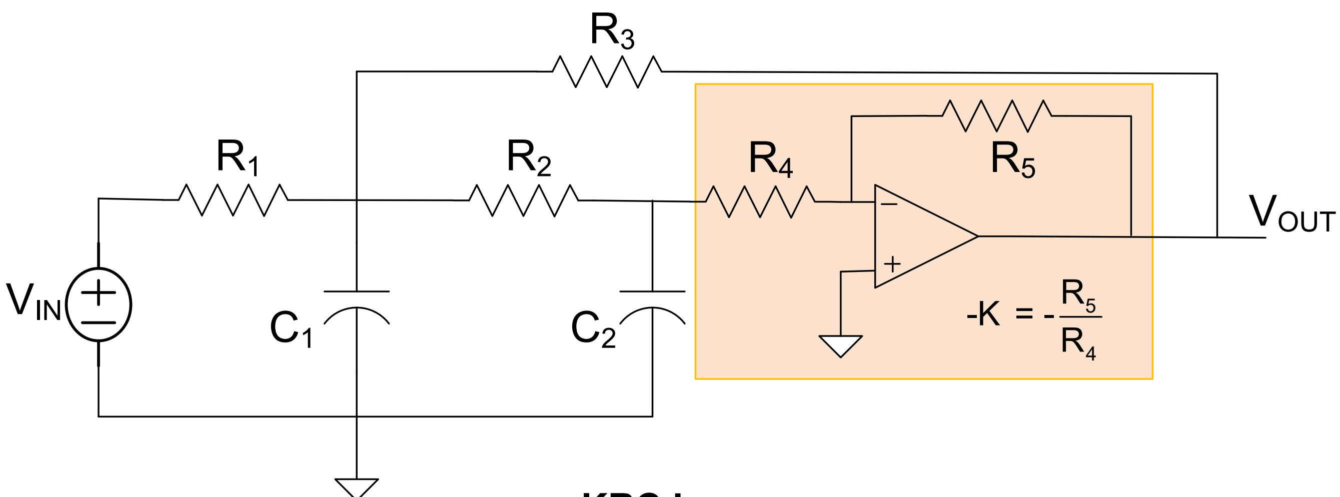

**Two-Integrator Loop**

(d)

# Sensitivity Comparisons

## Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)



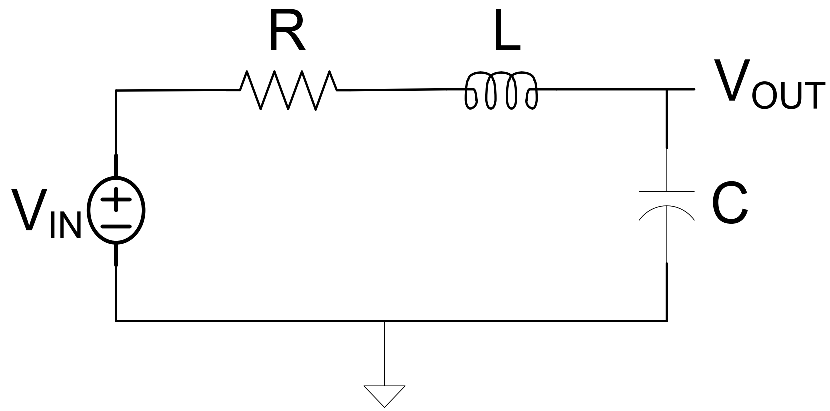

**-KRC Lowpass**

(e)

For all 5 structures, will have same transfer function within a gain factor

$$T(s) = \frac{K\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

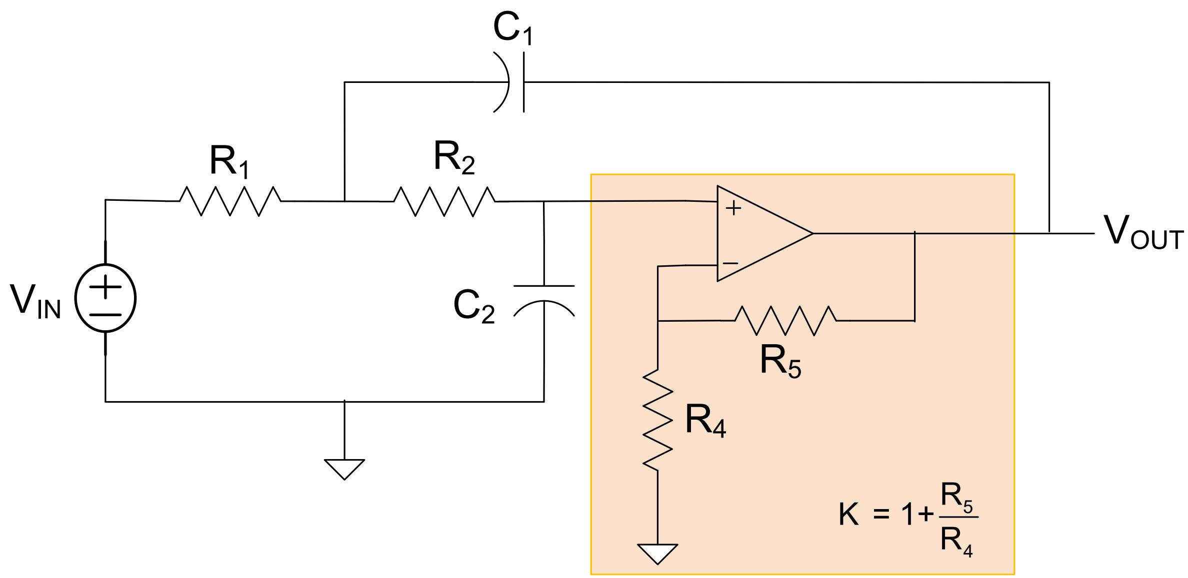
# a) – Passive RLC




$$T(s) = \frac{V_{OUT}}{V_{IN}} = \frac{1/LC}{s^2 + s\frac{R}{L} + 1/LC}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} \qquad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

# b) + KRC (a Sallen and Key filter)




$$T(s) = \frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2+s\left[\left(\dfrac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}}+\sqrt{\dfrac{R_2C_2}{R_1C_1}}+\sqrt{\dfrac{R_1C_2}{R_2C_1}}-K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)\right]+\dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\dfrac{R_1C_1}{R_2C_2}}+\sqrt{\dfrac{R_2C_2}{R_1C_1}}+\sqrt{\dfrac{R_1C_2}{R_2C_1}}-K\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)}$$

Case b1 : Equal R, Equal C

$$R_1 = R_2 = R \qquad C_1 = C_2 = C$$

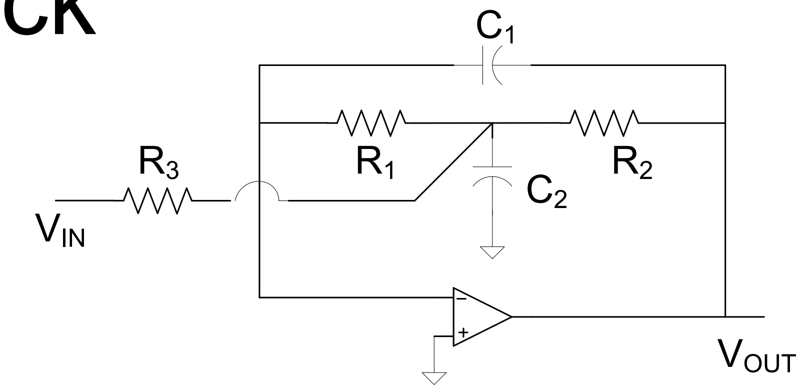
$$\omega_0 = \frac{1}{RC} \qquad K = 3 - \frac{1}{Q}$$

Case b2 : Equal R, K=1

$$R_1 = R_2 = R \qquad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}$$

$$T(s) = \frac{K\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

# c) Bridged T Feedback



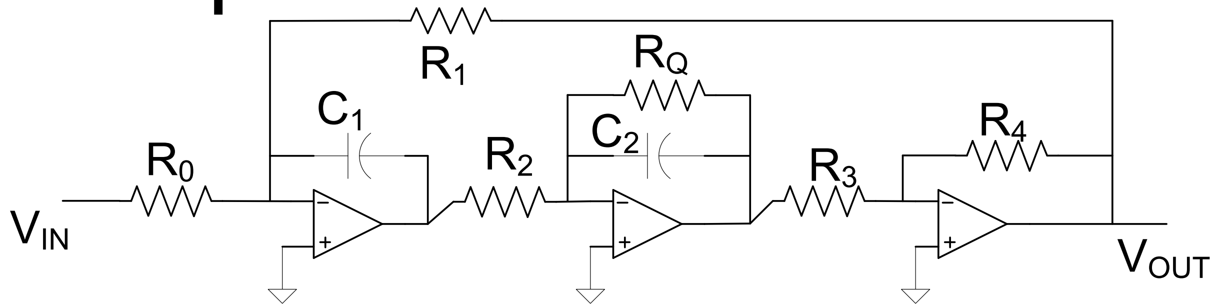

$$T(s) = \frac{\dfrac{1}{R_1R_3C_1C_2}}{s^2 + s\left[\left(\sqrt{\dfrac{C_2}{C_1}}\right)\left(\dfrac{1}{\sqrt{R_1R_2C_1C_2}}\right)\left(\sqrt{\dfrac{R_1}{R_3}} + \sqrt{\dfrac{R_2}{R_1}} + \dfrac{\sqrt{R_1R_2}}{R_3}\right)\right] + \dfrac{1}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1}{R_1R_2C_1C_2}} \qquad Q = \frac{1}{\left(\sqrt{\dfrac{C_2}{C_1}}\right)\left(\sqrt{\dfrac{R_1}{R_3}} + \sqrt{\dfrac{R_2}{R_1}} + \dfrac{\sqrt{R_1R_2}}{R_3}\right)}$$

If $R_1=R_2=R_3=R$ and $C_2=9Q^2C_1$

$$T(s) = \frac{\dfrac{1}{9Q^2R^2C_1^2}}{s^2 + s\left[\left(\dfrac{1}{3Q^2RC_1}\right)\right] + \dfrac{1}{9Q^2R^2C_1^2}}$$

# d) 2 integrator loop




$$T(s) = -\frac{\frac{R_4}{R_3} \bullet \frac{1}{R_0R_2C_1C_2}}{s^2 + s\left(\frac{1}{R_QC_2}\right) + \frac{R_4}{R_3} \bullet \frac{1}{R_0R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \bullet \frac{1}{R_0R_2C_1C_2}}$$

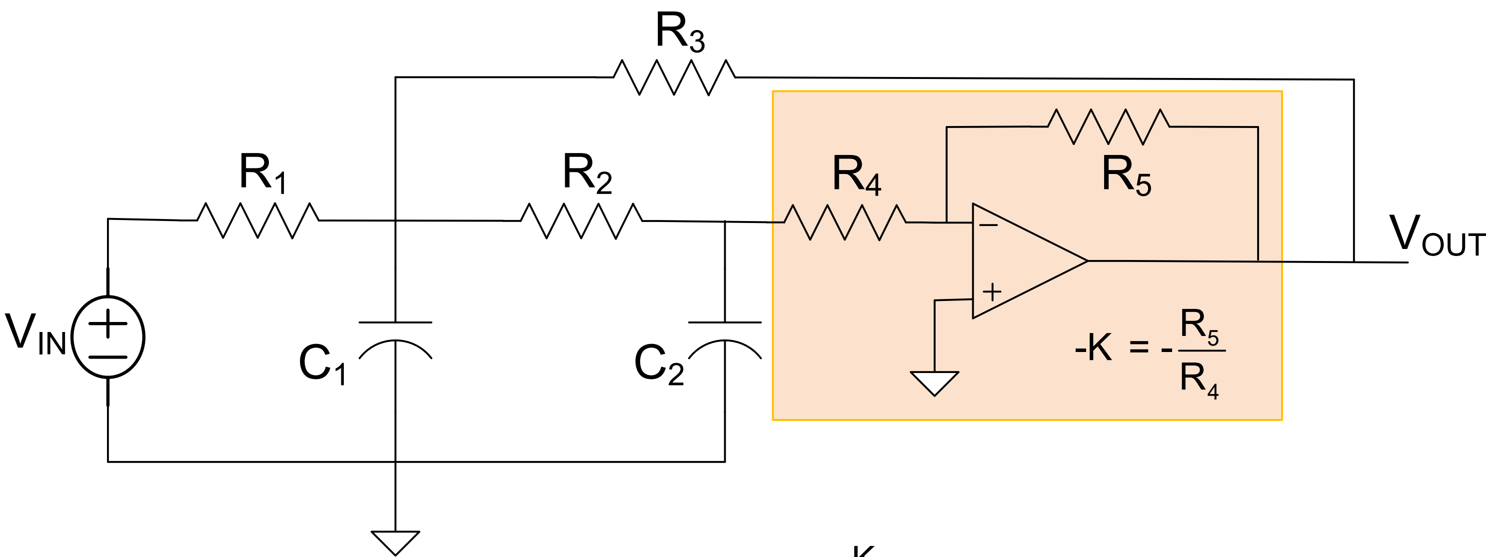
$$Q = \frac{R_Q}{\sqrt{R_0R_2}}\sqrt{\frac{C_2}{C_1}}$$

For: $R_0 = R_1 = R_2 = R$ $\quad C_1 = C_2 = C$ $\quad R_3 = R_4$

$$T(s) = -\frac{\frac{1}{R^2C^2}}{s^2 + s\left(\frac{1}{R_QC}\right) + \frac{1}{R^2C^2}}$$

$$R_Q = QR \qquad \omega_0 = \frac{1}{RC}$$

# d) - KRC (a Sallen and Key filter)




$$T(s) = -\frac{\dfrac{K}{R_1R_2C_1C_2}}{s^2+s\left[\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)\right]+\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$\omega_0 = \sqrt{\frac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\dfrac{1+(R_1/R_3)(1+K)+(R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}}{\left(1+\dfrac{R_1}{R_3}\right)\left(\dfrac{1}{R_1C_1}\right)+\left(1+\dfrac{C_2}{C_1}\right)\left(\dfrac{1}{R_2C_2}\right)+\left(\dfrac{1}{R_4C_2}\right)}$$

Often $R_1=R_2=R_3=R_4=R$, $C_1=C_2=C$

$$Q = \frac{\sqrt{5+K_0}}{5}$$

Stay Safe and Stay Healthy !

End of Lecture 22